



WORKING PAPERS

RESEARCH DEPARTMENT

WORKING PAPER NO. 07-27
INTEREST RATE VERSUS MONEY SUPPLY INSTRUMENTS:
ON THE IMPLEMENTATION OF MARKOV-PERFECT
OPTIMAL MONETARY POLICY

Michael Dotsey
Federal Reserve Bank of Philadelphia

Andreas Hornstein
Federal Reserve Bank of Richmond

September 26, 2007

RESEARCH DEPARTMENT, FEDERAL RESERVE BANK OF PHILADELPHIA

Ten Independence Mall, Philadelphia, PA 19106-1574 • www.philadelphiafed.org/econ/index.html

Interest Rate versus Money Supply Instruments: On the Implementation of Markov-Perfect Optimal Monetary Policy*

Michael Dotsey[†] and Andreas Hornstein[‡]

September 26, 2007

Abstract

Currently there is a growing literature exploring the features of optimal monetary policy in New Keynesian models under both commitment and discretion. With respect to time consistent policy, the literature focuses on solving for allocations. Recently, however, King and Wolman (2004) have examined implementation issues involved under time consistent policy when the monetary authority chooses nominal money balances. Surprisingly, they find that equilibria are no longer unique under a money stock regime. Indeed, there exist multiple steady states. We find that their conclusion of non-uniqueness of Markov-perfect equilibria is sensitive to the instrument of choice. If, instead, the monetary authority chooses the nominal interest rate rather than nominal money balances, there exists a unique Markov-perfect steady state and point-in-time equilibria are unique as well. Thus, in their language, monetary policy is implementable using an interest rate instrument while it is not implementable using a money stock instrument.

* We would like to thank Alex Wolman for many useful comments. The views expressed in this paper are those of the authors and do not necessarily represent those of the Federal Reserve Bank of Philadelphia, the Federal Reserve Bank of Richmond, or the Federal Reserve System. We can be reached at michael.dotsey@phil.frb.org and andreas.hornstein@rich.frb.org.

[†] Federal Reserve Bank of Philadelphia

[‡] Federal Reserve Bank of Richmond

1 Introduction

Currently there is a growing literature exploring the features of optimal monetary policy in New Keynesian models under both commitment and discretion. Generally, the literature focuses on solving for allocations. Recently, however, attention has been paid to analyzing more decentralized policies, and this paper is part of a growing investigation studying implementation of planning problems. We believe that this is an important area of inquiry, because the institutions responsible for setting policies rarely have direct control over allocations. It is, therefore important to understand whether or not a planner's allocations are obtainable with a given institutional structure.

For the case of time-consistent policies that are Markov-perfect, King and Wolman (2004) have examined implementation issues when the monetary authority uses a nominal money balances instrument. Surprisingly, they find that equilibria are no longer unique under a money-supply regime. Indeed, there exist multiple steady states. The non-uniqueness stems from strategic complementarities introduced through the policy process itself. For example, if agents believe that money and the price level will be high in the future, it is optimal for money and prices to be high today. Thus, expectations can be self-fulfilling and there exist more than one Markov-perfect equilibrium outcome.

In this paper we demonstrate that King and Wolman's (2004) result of a non-unique Markov-perfect equilibrium is sensitive to the choice of instrument. If the monetary authority chooses the nominal interest rate instead of nominal money balances, there exists a unique Markov-perfect steady state and point-in-time equilibria are unique as well. Essentially, using an interest rate instrument weakens strategic complementarities and restricts the money growth process in such a way that multiplicities of both steady state and point-in-time equilibria are ruled out. Simply put, choosing the interest rate constrains the anticipated evolution of the price level. Agents are not currently free to assume any future price level, and this constraint rules out the self-fulfilling equilibria that are present with a money-supply instrument. Thus, there is less scope for one firm's pricing strategy to influence another's.

These results, that money instruments yield a determinant equilibrium in rational expectations model, while interest-rate instruments do not, seemingly turn the results of Sargent and Wallace (1975) on their head. Note, however, that we are dealing with different uniqueness issues. Our approach, together with King and Wolman (2004), is concerned with the global uniqueness of the current period equilibrium conditional on future equilibrium outcomes. As such we are dealing with potential non-linearities in the competitive equilibrium characterization. Sargent and Wallace (1975), on the other hand, are concerned with the local uniqueness

of the linear approximation of the dynamic rational expectations equilibrium. For the case of an interest rate policy, our restriction to Markov-perfect equilibria picks one equilibrium out of the continuum of possible rational expectations equilibria.¹ The restriction to Markov-perfect equilibria is then analogous to McCallum's (1983) minimal state variable solution.

The paper proceeds as follows. First we briefly describe a standard New-Keynesian economy, which is identical to the one used by King and Wolman (2004). We then explore time-consistent Markov-perfect policy. We briefly review the King and Wolman (2004) result and then investigate how using an interest rate instrument overturns the conclusions of their analysis. We note that a full decentralization cannot be obtained if the equilibrium concept is Markov perfect. A brief summary concludes.

2 The economy

There is an infinitely lived representative household with preferences over consumption and leisure. The consumption good is produced using a constant-returns-to-scale technology with a continuum of differentiated intermediate goods. Each intermediate good is produced by a monopolistically competitive firm with labor as the only input. Intermediate goods firms set the nominal price for their products for two periods, and an equal share of intermediate firms adjusts their nominal price in any particular period. Also, in what follows we restrict our analysis to perfect foresight economies.

2.1 The representative household

The representative household's utility is a function of consumption c_t , and the fraction of time spent working n_t ,

$$\sum_{t=0}^{\infty} \beta^t [\ln c_t - \chi n_t], \quad (1)$$

where $\chi \geq 0$, and $0 < \beta < 1$. The household's period budget constraint is

$$P_t c_t + B_t + M_t \leq W_t n_t + R_{t-1} B_{t-1} + M_{t-1} + V_t + T_t, \quad (2)$$

where P_t is the nominal price level, W_t is the nominal wage rate, B_t and M_t are the end-of-period holdings of nominal bonds and money, T_t are lump-sum transfers, and R_{t-1} is the gross nominal interest rate on bonds. The agent owns all firms in the economy, and V_t is

¹Giannoni and Woodford (2002a, 2002b) have investigated the local implementation of allocations originating from optimal policy under full commitment.

nominal profit income from firms. The household is assumed to hold money in order to pay for consumption purchases

$$M_t = P_t c_t. \quad (3)$$

We will use the term “real” to denote nominal variables deflated by the nominal price level, which is the price of the aggregate consumption good, and we use lower case letters to denote real variables. For example, real balances are $m_t \equiv M_t/P_t$.

The relevant first order conditions of the representative household’s problem are

$$1/c_t = \lambda_t \quad (4)$$

$$w_t/c_t = \chi, \quad (5)$$

$$1 = \beta \frac{\lambda_{t+1}}{\lambda_t} \cdot \frac{R_t}{P_{t+1}/P_t}, \quad (6)$$

Equation (4) equates the multiplier on the households budget constraint, λ , with the marginal utility of consumption. Equation (5) states that the marginal utility derived from the real wage equals the marginal disutility from work. Equation (6) is the Euler equation, and states that if the real rate of return increases, then the household increases future consumption relative to today’s consumption.

2.2 Firms

The consumption good is produced using a continuum of differentiated intermediate goods as inputs to a constant-returns-to-scale technology. Producers of the consumption good behave competitively in their markets. There is a measure one of intermediate goods, indexed $j \in [0, 1]$. Production of the consumption good c as a function of intermediate goods $y(j)$ is

$$c_t = \left[\int_0^1 y_t(j)^{(\varepsilon-1)/\varepsilon} dj \right]^{\varepsilon/(\varepsilon-1)} \quad (7)$$

where $\varepsilon > 1$. Given nominal prices $P(j)$ for the intermediate goods, the nominal unit cost and price of the consumption good is

$$P_t = \left[\int_0^1 P_t(j)^{1-\varepsilon} dj \right]^{1/(1-\varepsilon)}. \quad (8)$$

For a given level of production, the cost-minimizing demand for intermediate good j depends on the good’s relative price, $p(j) \equiv P(j)/P$,

$$y_t(j) = p_t(j)^{-\varepsilon} c_t. \quad (9)$$

Each intermediate good is produced by a single firm, and j indexes both the firm and good. Firm j produces $y(j)$ units of its good using a constant-returns technology with labor as the only input,

$$y_t(j) = n_t(j). \quad (10)$$

Each firm behaves competitively in the labor market, and takes wages as given. Real marginal cost in terms of consumption goods is

$$\psi_t = w_t. \quad (11)$$

Since each intermediate good is unique, intermediate goods producers have some monopoly power, and they face downward sloping demand curves (9). Intermediate goods producers set their nominal price for two periods, and they maximize the discounted expected present value of current and future profits:

$$\max_{P_t(j)} \left[\frac{P_t(j)}{P_t} - \psi_t \right] y_t(j) + \beta \frac{\lambda_{t+1}}{\lambda_t} \left[\frac{P_t(j)}{P_{t+1}} - \psi_{t+1} \right] y_{t+1}(j). \quad (12)$$

Since the firm is owned by the representative household, the household's intertemporal marginal rate of substitution is used to discount future profits. Using the definition of the firm's demand function (9) and the household's intertemporal rate of substitution, the first order condition for profit maximization can be written as

$$\left[\frac{P_t(j)}{P_t} \right]^{1-\varepsilon} \left[1 - \frac{\mu \psi_t}{P_t(j)/P_t} \right] + \beta \left[\frac{P_t(j)}{P_{t+1}} \right]^{1-\varepsilon} \left[1 - \frac{\mu \psi_{t+1}}{P_t(j)/P_{t+1}} \right] = 0 \quad (13)$$

where $\mu = \varepsilon/(\varepsilon - 1)$ is the static markup with flexible prices.

2.3 A symmetric equilibrium

We will assume a symmetric equilibrium; all firms that face the same constraints behave the same. This means that in every period there will be two firm types: the firms that adjust their nominal price in the current period, type 0 firms with relative price p_0 , and the firms that adjusted their price in the previous period, type 1 firms with current relative price p_1 . The current relative price $p_{1,t}$ is related to last period's relative price $p_{0,t-1}$ through the inflation rate $\pi_t \equiv P_t/P_{t-1}$

$$p_{1,t} = p_{0,t-1}/\pi_t. \quad (14)$$

Each period half of all firms have the option to adjust their nominal price.

The equilibrium of the economy is completely described by the sequence of marginal cost, relative prices, inflation rates, nominal interest rates, and real balances, $\{\psi_t, p_{0,t}, p_{1,t}, \pi_t, R_t, m_t\}$,

such that

$$1 = \frac{1}{2} [p_{0,t}^{1-\varepsilon} + p_{1,t}^{1-\varepsilon}] \quad (15)$$

$$0 = (p_{0,t})^{1-\varepsilon} \left(1 - \mu \frac{\psi_t}{p_{0,t}}\right) + \beta (p_{1,t+1})^{1-\varepsilon} \left(1 - \mu \frac{\psi_{t+1}}{p_{1,t+1}}\right) \quad (16)$$

$$\pi_{t+1} = \frac{p_{0,t}}{p_{1,t+1}} \quad (17)$$

$$\psi_t = \frac{\pi_{t+1}}{\beta R_t} \psi_{t+1} \quad (18)$$

$$m_t = \psi_t / \chi \quad (19)$$

2.4 Distortions and optimal policy

Allocations in this economy are suboptimal because of two distortions. The first distortion results from the monopolistically competitive structure of intermediate goods productions: the price of an intermediate good is not equal to its marginal cost. The average mark-up in the economy is the inverse of the real wage, P/W , which is, according to equation (11), the inverse marginal cost $1/\psi$. The second distortion reflects inefficient production when relative prices are different from one. Using the firm's demand function (9) and aggregate production (7) we can obtain the total demand for labor as a function of relative prices and aggregate output. Solving aggregate labor demand for aggregate output we obtain an 'aggregate' production function

$$c = an \text{ with } a \equiv 2/[p_0^{-\varepsilon} + p_1^{-\varepsilon}], \quad (20)$$

where for ease of exposition we will drop time subscripts when possible and denote next period's values by a prime. Given the symmetric production structure, equations (7) and (10), efficient production requires that equal quantities of each intermediate good are produced. The degree of allocational inefficiency is reflected in the term $a \leq 1$. The allocation is efficient if $p_0 = p_1 = a = 1$.

The policymaker is assumed to maximize lifetime utility of the representative agent, taking the competitive equilibrium conditions (15)-(19) as constraints. For a time-consistent Markov-perfect policy it is furthermore assumed that the policymaker takes future policy choices as given and that policy choices are functions of pay-off relevant state variables only. Because, there are no state variables in our example this amounts to the planner maximizing the current period utility function of a representative agent and choosing an unconditional value for the policy instrument. Taking future policy as given means that the planner has no control over future outcomes such as future relative prices or allocations.

Typically, one states the problem in terms of the planner choosing the competitive equilibrium allocation. Alternatively, the planner chooses the relative price p_0 . The choice of p_0

determines p_1 , equation (15), and allocational efficiency, equation (20). The choice of p_0 also determines marginal cost, equation (16), and thereby consumption, equations (5) and (11). Finally, given consumption and allocational efficiency employment is determined, equation (20). In this model, with $\varepsilon = 11$, implying a markup of approximately 10 percent, and $\chi = 1/1.1$ the optimal allocation consumption of consumption and labor is .9996 and 1.0 respectively. Thus, there is very little allocational inefficiency. This allocation implies an inflation rate of 1.82 percent and a nominal interest rate of 2.84 percent.

3 A Markov-perfect money supply policy

King and Wolman (2004) show that decentralizing the planning problem so that the planner chooses a policy rule based on money leads to problems. In particular, they assume a homogeneous monetary policy rule that sets the nominal money stock in proportion to the preset nominal price from the last period

$$M = \tilde{m}P_1 \quad (21)$$

Combining the policy rule (21) with the money demand equation (3) yields the modified policy rule in real terms

$$c = \tilde{m}p_1 \quad (22)$$

Finally, combining (22) with the optimal labor supply condition (5) yields the equilibrium condition for marginal cost

$$\psi = \chi\tilde{m}p_1. \quad (23)$$

We now revisit the King and Wolman (2004) result, which indicates that for most values of the money-supply policy parameter, \tilde{m} , the steady state will not be unique. Since in a Markov-perfect equilibrium without state variables the expected future policy has to be a steady state, non-uniqueness of the steady state alone suggests that the monetary policy rule may result in indeterminacy of the competitive equilibrium.

Lemma 1 *There exist values $\tilde{m}_{\min} < \tilde{m}^* = 1/(\chi\mu) \leq \tilde{m}^{**}$ such that (1) if $\tilde{m} \in (\tilde{m}_{\min}, \tilde{m}^*]$ then there exists a unique non-inflationary steady state; (2) if $\tilde{m} \in (\tilde{m}^*, \tilde{m}^{**})$, then there exist two inflationary steady states; (3) if $\tilde{m} = \tilde{m}^{**}$ then there exists a unique inflationary steady state; and (4) if $\tilde{m} > \tilde{m}^{**}$ then no steady state exists.*

Proof. Substitute (23) for marginal cost in (16) and obtain the following steady state mapping from the inflation rate to the policy parameter

$$\tilde{m} = \frac{1}{\chi\mu}h(\pi) \text{ and } h(\pi) = \frac{1 + \beta\pi^{\varepsilon-1}}{1/\pi + \beta\pi^{\varepsilon-1}}.$$

In steady state, the nominal interest rate, $R \geq 1$, and because $\beta R = \pi$, $\pi \geq \beta$. For $\pi \in (\beta, 1]$, $h(\pi)$ is strictly increasing and less than one. For positive inflation, $\pi > 1$, the function h satisfies (1) $h(\pi) > h(1) = 1$ and (2) $h(\infty) = 1$. Since h is continuous the function must eventually be decreasing if it is to approach 1 as $\pi \rightarrow \infty$. So there must exist an inflation rate π^{**} such that $h(\pi) \leq h^{**} = h(\pi^{**})$. Let $\tilde{m}^* = 1/(\chi\mu)$ and $\tilde{m}^{**} = \tilde{m}^* h^{**}$. The lemma follows immediately from the properties of the h function. ■

Figure 1 displays the steady state inflation rates π consistent with the money rule \tilde{m} for the parameter values $\beta = 0.99$, $\varepsilon = 11$, and $\chi = 1/1.1$. Note that \tilde{m}^* is the steady state money-supply policy parameter associated with a constant price level.

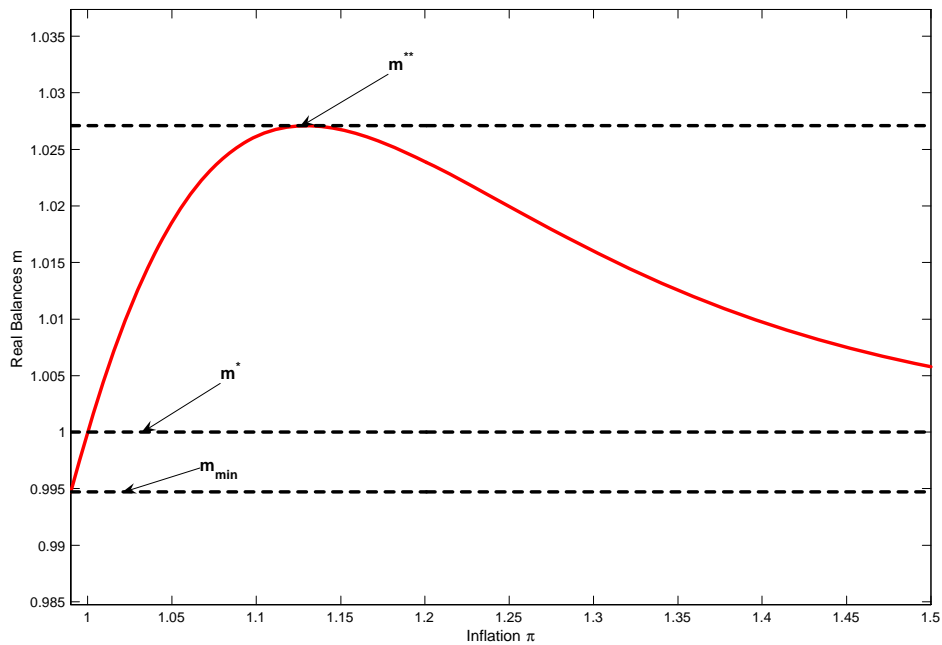


Figure 1. Steady State Multiplicity with a Money Rule

King and Wolman (2004) discuss the presence of strategic complementarities that arise under time-consistent policy. If price-setting firms expect that the policymaker will try to implement an outcome with positive inflation, then because of the strategic complementarities, two different levels for the optimal relative price and therefore two different levels of marginal cost are possible equilibrium outcomes. This in turn implies that any choice of the money supply instrument that tries to implement a positive inflation rate cannot uniquely determine the inflation rate: in general, two distinct outcomes are possible. We will see that this result is different from the case of an interest rate rule which essentially fixes the expected inflation rate, and in turn marginal cost.

We now define a reaction function that describes how an individual firm sets its price in response to anticipated aggregate outcomes. For this purpose combine (16) and (17) and rewrite the optimal pricing equation as

$$p_0 = \mu \frac{\psi + \beta \psi' \pi'^\varepsilon}{1 + \beta \pi'^{\varepsilon-1}}. \quad (24)$$

Primes denote next period's values. The left-hand side of this expression is the optimal relative price chosen by a particular price adjusting firm conditional on the expected inflation rate. The expected inflation rate, however, does depend on the pricing decisions of all other firms (17). King and Wolman (2005) argue that for the above introduced money supply rule the reaction function exhibits strategic complementarities: if other firms decide to set a higher relative price, it is in the interest of an individual firm to also set a higher relative price. Strategic complementarities in turn may result in equilibrium indeterminacy.

A simple illustration of King and Wolman' result, that there are multiple equilibria for a money stock rule, can be obtained by looking at the case when both current and future policymakers choose the same policy rule, $\tilde{m} = \tilde{m}' \in (\tilde{m}^*, \tilde{m}^{**})$. This policy is consistent with the existence of two steady state equilibria. Furthermore, conditional on choosing future behavior to be in accord with one of the two possible steady states, there exist two equilibria in the current period. Together with the policy rule (23) and the definition of the inflation rate (17), the reaction function simplifies to

$$\frac{1}{\mu \chi \tilde{m}} p_0 = \bar{p}_0 \frac{(\bar{p}_1 / \bar{p}_0) + \beta (\bar{p}_0 / \bar{p}_1')^{\varepsilon-1}}{1 + \beta (\bar{p}_0 / \bar{p}_1')^{\varepsilon-1}} = g(\bar{p}_0, \bar{p}_1'), \quad (25)$$

where an overbar indicates an aggregate variable.

Note that equation (15) provides a unique solution for the relative price of the firm that cannot adjust its price, conditional on the price of the firm that can adjust its price,

$$p_1(p_0) = (2 - p_0^{1-\varepsilon})^{1/(1-\varepsilon)} \text{ for } p_0 > \underline{p}_0 = (1/2)^{1/(\varepsilon-1)} < 1. \quad (26)$$

It then follows that p_1 is a decreasing function of p_0 , and $p_1 \left\{ \begin{smallmatrix} \leq \\ \geq \end{smallmatrix} \right\} 1$ when $p_0 \left\{ \begin{smallmatrix} \geq \\ \leq \end{smallmatrix} \right\} 1$. This in turn implies that the g -term of the reaction function in terms of the relative price p_0 intersects the 45-degree line at $p_0 = 1$, and is above (below) the 45-degree line when p_0 is less than (greater than) one, equation (27). As \bar{p}_0 becomes large g converges to the 45-degree line from below, equation (28).

$$g(\bar{p}_0, \bar{p}_1') \left\{ \begin{smallmatrix} < \\ = \\ > \end{smallmatrix} \right\} \bar{p}_0 \text{ for } \bar{p}_0 \left\{ \begin{smallmatrix} > \\ = \\ < \end{smallmatrix} \right\} 1 \quad (27)$$

$$\lim_{\bar{p}_0 \rightarrow \infty} g(\bar{p}_0, \bar{p}_1') = \bar{p}_0, \quad (28)$$

$$\frac{\partial g}{\partial p_0} \Big|_{\overline{p_0}=1} = -\frac{1 - \beta (\overline{p_1}')^{1-\varepsilon}}{1 + \beta (\overline{p_1}')^{1-\varepsilon}} \quad (29)$$

Finally equation (29) can be shown with some additional algebra.

It is thus easily shown that for $\tilde{m} \in (\tilde{m}^*, \tilde{m}^{**})$ the LHS and the RHS. of expression (25) will in general intersect twice. On the one hand, from Lemma 1 it follows that since $\tilde{m} > \tilde{m}^*$, the constant term in equation (25) is greater than one, $\mu\chi\tilde{m} > 1$. Thus the LHS. defines a line through the origin with slope less than one, that is, below the 45-degree line. On the other hand, the RHS. of (25) intersects the 45-degree line at $p_0 = 1$, and stays above (below) the 45-degree line whenever p_0 is less than (greater than) one. Furthermore, as p_0 becomes arbitrarily large the RHS. of (25) converges to the 45-degree line from below.

Since the LHS. is strictly below the RHS. for $p_0 \leq 1$, the two curves do not intersect in this range. We know that at least one intersection point exists since we consider policy rules that are consistent with the existence of a steady state, and the steady state price is a solution to the reaction function (25). Thus there must be an intersection point for $p_0 > 1$.

If $\tilde{m} = \tilde{m}^*$, then we know that a unique non-inflationary steady state with $p_0 = 1$ exists, and this steady state also satisfies (25). For this case, the LHS. is the 45-degree line and the RHS. has a unique intersection with the 45-degree line at $p_0 = 1$. Furthermore, from (29) it follows that the slope of the RHS. at $p_0 = 1$ is negative. With a marginally larger value of \tilde{m} , the slope of the LHS. becomes less than one, and there will be at least two intersection with the RHS. to the right of $p_0 = 1$.

Figure 2 displays the reaction function for the money rule conditional on the parameterization used in Figure 1 and assuming that next period's policy generates a steady state inflation rate $\pi^* = 1.05$. We summarize our discussion in the following lemma.

Lemma 2 *Suppose the current and future policymakers use the same money stock rule \tilde{m} . If $\tilde{m} \in (\tilde{m}^*, \tilde{m}^{**})$, then, in general, at least two competitive equilibria exist. If $\tilde{m} = \tilde{m}^*$ then the competitive equilibrium is unique.*

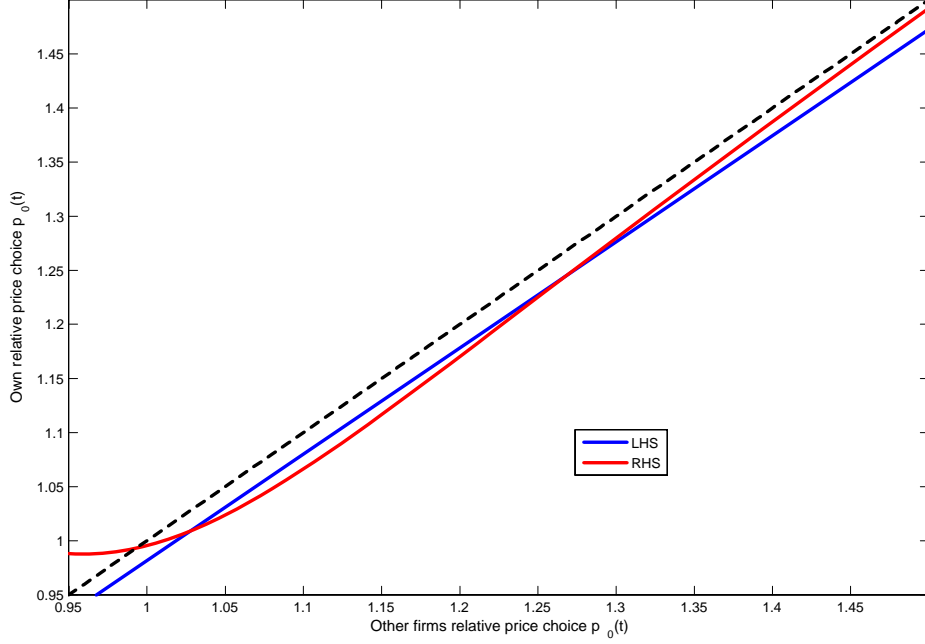


Figure 2. Reaction function for money rule

4 A Markov-perfect interest rate policy

In this section we perform a careful evaluation of the benefits of using an interest rate instrument for Markov-perfect policies. Even though we find policy-induced strategic complementarities, we find that steady states and point in time equilibria are unique. In what follows, we solve for the current equilibrium conditional on current policy R and future equilibrium outcomes $Y' = (p'_0, p'_1, \psi')$. To begin, we show that the equilibrium reaction function again exhibits strategic complementarities.

With a fixed nominal interest rate policy affects marginal cost through the Euler equation,

$$\psi = \frac{\psi'}{\beta R p'_1} p_0 \quad (30)$$

which combines (17) and (18). Substituting the marginal cost equation (30) into the optimal price setting equation (24) yields the reaction function

$$\frac{p'_1}{\mu \psi'} p_0 = p_0 \frac{1/(\beta R) + \beta (p_0/p'_1)^{\varepsilon-1}}{1 + \beta (p_0/p'_1)^{\varepsilon-1}} = p_0 h(p_0, p'_1). \quad (31)$$

This reaction function also exhibits strategic complementarities, that is, the right-hand side of equation (31) is increasing in p_0 , but these strategic complementarities are not strong enough

to induce multiple competitive equilibria.² If we compare (25) and (31) we see that in terms of their dependence on the price chosen by price adjusting firms the two reaction functions differ only in the first term of the numerator. Whereas for the money stock rule the relative price of goods declines as the price of the price adjusting firms increases, this term remains constant for the interest rate rule. Compared with the money rule reaction function this means that even for low values of p_0 the interest rate rule reaction function is sufficiently steep to prevent the occurrence of multiple intersections of the LHS and RHS expressions. Figure 3 displays the reaction function for the interest rate policy conditional on the parameterization used in Figure 1 and assuming that next period's policy generates a steady state inflation rate $\pi^* = 1.05$.

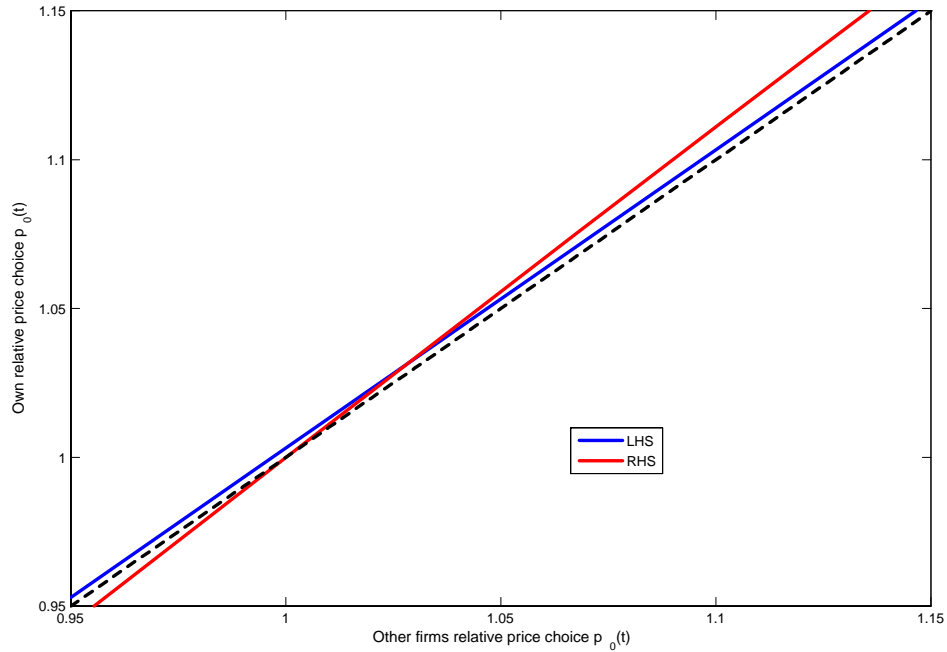


Figure 3. Reaction function for interest rate rule

In order to prove existence and uniqueness of a (steady state) competitive equilibrium we do not study the reaction function directly, but rewrite the optimal pricing equation (16) as

$$(p_0)^{1-\varepsilon} \left(1 - \mu \frac{\psi}{p_0} \right) = -\beta (p'_1)^{1-\varepsilon} \left(1 - \mu \frac{\psi'}{p'_1} \right) = -\beta A' \quad (32)$$

Conditional on next period's equilibrium outcome (p'_1, ψ') , equation (30) and (32) determine current marginal cost ψ and the profit maximizing choice for the relative price p_0 .

²One can show that the h function is increasing in p_0 if $R > 1/\beta$; that is, the policy is inflationary.

We will first demonstrate the existence of a unique steady state conditional on a given nominal interest rate. We then show that for any current interest rate there exists a unique equilibrium if future expected monetary policy is inflationary or deflationary and equilibrium indeterminacy occurs only if the expected future monetary policy maintains stable prices.

Lemma 3 *Conditional on the nominal interest rate $R \geq 1$, there exists a unique steady state $p_0^* = p_0 = p_0'$, $p_1^* = p_1 = p_1'$, $\psi^* = \psi = \psi'$.*

Proof. Equation (30) determines the unique steady state inflation rate

$$\pi^* = p_0^*/p_1^* = \beta R. \quad (33)$$

Equations (15) and (33) uniquely determine the steady state relative prices (p_0^*, p_1^*) . From equation (16) we obtain the steady state marginal cost

$$\psi^* = \frac{1}{\mu} \frac{1 + \beta (\pi^*)^{\varepsilon-1}}{1 + \beta (\pi^*)^{\varepsilon}} p_0^*. \quad (34)$$

■

For the special case of the non-inflationary steady state with stable prices, $\pi^* = 1$, there are no production inefficiencies, but prices exceed marginal cost. From (15) and (33) we have that

$$p_0^* = p_1^* = 1. \quad (35)$$

From (34) it then follows that marginal cost is equal to the inverse static markup

$$\psi^* = 1/\mu. \quad (36)$$

We now show that a zero inflation steady state cannot be implemented as a unique equilibrium under an interest rate instrument.

Lemma 4 *If next period's policy choice attains a steady state outcome with stable prices, then (1) the current period equilibrium is indeterminate if current policy also tries to attain the stable-price steady state $\beta R = 1$; (2) no current period equilibrium exists if $\beta R \neq 1$.*

Proof. The current period equilibrium is defined by equations (30) and (32) where next period's variables are evaluated at their steady state values (35) and (36):

$$\psi = \frac{p_0}{\beta R \mu} \text{ and } (p_0)^{1-\varepsilon} \left(1 - \mu \frac{\psi}{p_0} \right) = 0. \quad (37)$$

Now, if current policy also targets stable prices, $\beta R = 1$, then (37) is satisfied for any feasible combination of (p_0, ψ)

$$p_0 > \underline{p}_0 \text{ and } \psi = p_0/\mu.$$

If current policy is inflationary or deflationary, $\beta R \neq 1$, then the only solution to (37) is $p_0 = 0$. But $p_0 = 0$ is not a feasible outcome, so no equilibrium exists. ■

Lemma 5 *If next period's policy choice attains an inflationary or deflationary steady state outcome, then (1) for any nominal interest rate for which a current period equilibrium exists it is unique, and (2) there always exists a nominal interest rate for which an equilibrium exists.*

Proof. The current equilibrium is defined by the two equations (30) and (32) which map the current period relative price p_0 to current period marginal cost ψ . Rewriting (32), we have the following two equations:

$$\begin{aligned} \psi &= f_1(p_0) = \left(\frac{1}{\beta R} \frac{\psi'}{p_1'} \right) p_0 \\ \psi &= f_2(p_0) = \frac{1}{\mu} (p_0 + \beta A' p_0^\varepsilon). \end{aligned}$$

An intersection of the two functions represents a potential equilibrium. The two functions always intersect at $p_0 = 0$, but $p_0 = 0$ is not a feasible outcome. Both functions are strictly increasing at $p_0 = 0$,

$$\begin{aligned} \frac{\partial f_1}{\partial p_0} &= \frac{1}{\mu} \frac{1 + \beta (\pi')^{\varepsilon-1}}{1 + \beta (\pi')^\varepsilon} \frac{1}{\beta R} \\ \frac{\partial f_2}{\partial p_0} &= \frac{1}{\mu} (1 + \beta A' \varepsilon p_0^{\varepsilon-1}). \end{aligned}$$

The function f_2 is strictly concave (convex) if $A' < 0$ ($A' > 0$),

$$\frac{\partial^2 f_2}{\partial p_0^2} = \frac{1}{\mu} \beta A' \varepsilon (\varepsilon - 1) p_0^{\varepsilon-2}$$

The sign of the term A' depends on the inflationary stance of next period's steady state policy. From (16) we get

$$\begin{aligned} \beta A' &= \beta (p_1')^{1-\varepsilon} \left\{ 1 - \mu \left[\frac{1}{\mu} \frac{1 + \beta (\pi')^{\varepsilon-1}}{1 + \beta (\pi')^\varepsilon} p_0' \right] \frac{1}{p_1'} \right\} \\ &= \beta (p_1')^{1-\varepsilon} \left\{ 1 - \pi' \frac{1 + \beta (\pi')^{\varepsilon-1}}{1 + \beta (\pi')^\varepsilon} \right\} \\ &= \beta (p_1')^{1-\varepsilon} \left\{ \frac{1 - \pi'}{1 + \beta (\pi')^\varepsilon} \right\}. \end{aligned}$$

The first equality uses the steady state expression for next period's marginal cost (34), and the second equality uses the steady state expression for next period's inflation rate (33). Thus A' is negative if next period's policy is inflationary, $\pi' > 1$, and A' is positive if the policy is deflationary, $\pi' < 1$.

Since the function f_1 is linear and the function f_2 is strictly concave or convex, if an intersection between f_1 and f_2 exists for positive values of p_0 , it is unique. Suppose that next period's policy is inflationary, that is, the function f_2 is concave. Then the two functions intersect for positive p_0 if at $p_0 = 0$ the function f_2 is steeper than f_1 ,

$$\frac{\partial f_1}{\partial p_0} = \frac{1}{\mu} \frac{1}{\beta R} \frac{\pi' + \beta (\pi')^\varepsilon}{1 + \beta (\pi')^\varepsilon} < \frac{1}{\mu} = \frac{\partial f_2}{\partial p_0} \Big|_{p_0=0}$$

This condition can always be satisfied for a sufficiently large nominal interest rate $R \geq 1$. In other words the policymaker can always find an interest rate for which the functions intersect. Recall that there is a lower bound for feasible relative prices p_0 , equation (26), so the policymaker has to choose an interest rate that implies a sufficiently large value for the relative price p_0 . A policymaker can always find such an interest rate, since he can always replicate the steady state by choosing $R = R'$. Thus there exists a choice for R such that an equilibrium exists and it is unique. An analogous argument applies if next period's policy is deflationary. ■

5 Conclusion

This paper has analyzed the importance of the monetary policy instrument in decentralizing a time-consistent planner's optimal policy. In that regard, it is part of growing literature investigating the implementation of optimal plans. We have shown that whether a planner uses a money instrument or an interest-rate instrument is crucial for determining if optimal Markov-perfect allocations can be attained via the appropriate setting of the instrument. King and Wolman (2004) were the first to alert us to the nontrivial ramifications of decentralization. They produced a surprising result of significant impact, namely that decentralization is a non-trivial problem and with regard to using a money instrument implementation of the optimal allocation is unattainable. A time-consistent planner using a money instrument could not achieve the allocations chosen by a planner who was able to directly pick allocations. In fact, they showed that steady states and equilibria were not unique at the optimal inflation rate. Since, in reality, no central bank picks allocations this result presented a challenge for understanding just how a time-consistent central bank might operate.

Intuition gained from the early rational expectations literature on monetary policy as depicted in Sargent and Wallace (1975) would suggest that an interest rate instrument would

have similar problems. Here we have shown that it does not. A planner using an interest-rate instrument can achieve the Markov-perfect allocations of the standard time-consistent planning problem. The result occurs for two key reasons. The interest rate instrument pins down future inflation in ways unobtainable using a money instrument, and in so doing reduces the degree of strategic complementarity that arises from the time-consistent policy problem itself.

References

- [1] Dotsey, Michael and Andreas Hornstein, 2003 "Should a Monetary Policy Maker Look at Money," *Journal of Monetary Economics*, 50 no 3, (April), 547-580.
- [2] Giannoni, Marc and Michael Woodford, (2002a), "Optimal Interest-Rate Rules I: General Theory," NBER working paper 9419.
- [3] Giannoni, Marc and Michael Woodford, (2002b), "Optimal Interest-Rate Rules II: Applications," NBER working paper 9420.
- [4] King, R. G., A.L. Wolman, (2004.) "Monetary Discretion, Pricing Complementarity, and Dynamic Multiple Equilibria," *Quarterly Journal of Economics*, vol. 119 (November), 1513-1553.
- [5] McCallum, Bennett T. (1983) "On Non-Uniqueness in Rational Expectations Models: An Attempt at Perspective," *Journal of Monetary Economics*, vol 11 (March), 139-168.
- [6] Sargent, Thomas and Neil Wallace, (1975), "Rational Expectations, the Optimal Monetary Instrument, and the Optimal Money Supply Rule," *Journal of Political Economy*, 83 (April), 242-254. .
- [7] Wolman, A. L. (2001). "A Primer on Optimal Monetary Policy with Staggered Price-Setting," *Federal Reserve Bank of Richmond Economic Quarterly* 87 (Fall), 27-52.